PERIOD-III

M12CH11

TOPIC

11 Trigonometry-2

11.1. GRAPH OF TRIGONOMETRIC FUNCTIONS

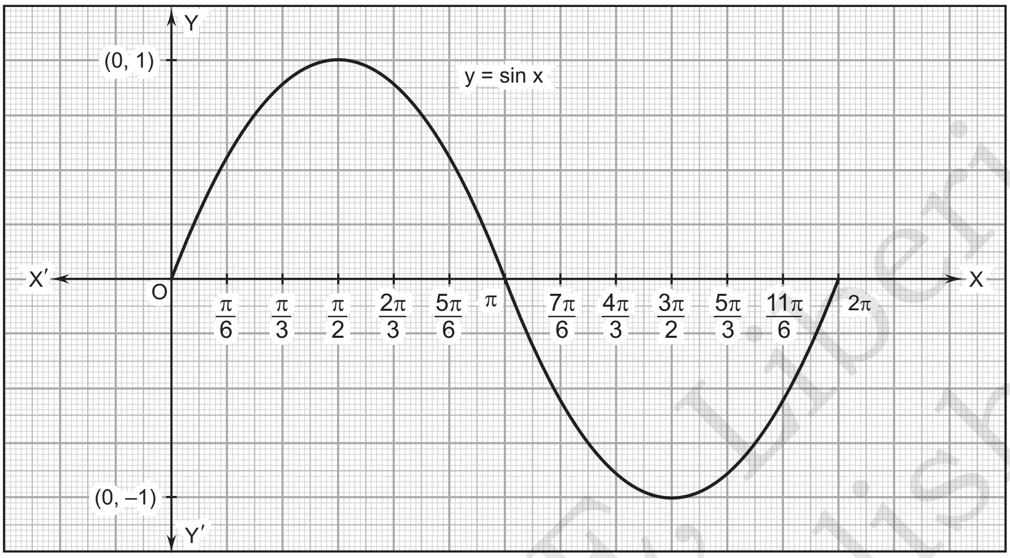
1. Graph of Sine Function

We know that $f(x)$ = sin x is a periodic function with period 2π. Therefore, it is sufficient to know graph $f(x)$ = sin x in the interval $[0, 2\pi]$. Using the periodicity of the function, we can draw the graph of $f(x)$ = sin x in other intervals such as $[-2\pi, 4\pi]$ etc.

Border to draw the graph of $f(x)$ = sin x in the interval $[0, 2\pi]$, we require the values of sin x at these points in $[0, 2\pi]$. These values are listed in the following table.

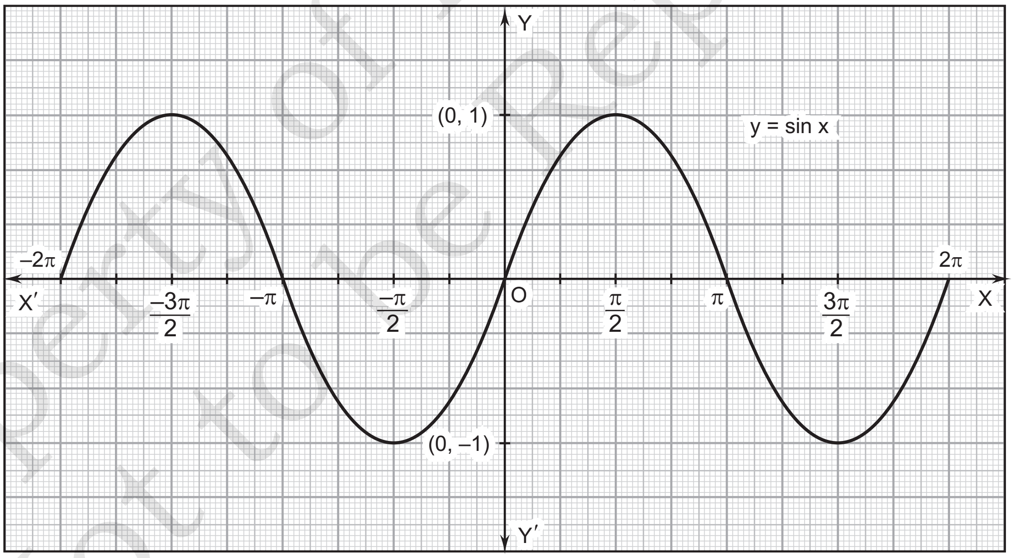
x	0	$\frac{\pi}{12}$	$\frac{\pi}{8}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{3\pi}{8}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$
	$\frac{7\pi}{12}$	$\frac{5\pi}{8}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{7\pi}{8}$	$\frac{11\pi}{12}$	2π	
y = sin x	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0	
y = sin α	0	0.5	0.71	0.86	1	0.86	0.71	0.5	0
	–0.5	–0.71	–0.86	–1	–0.86	–0.71	–0.5	0	

On suitable scale we plot the point (0, 0) $(\pi/6, 0.5)$, $(\pi/4, 0.71)$, $(\pi/3, 0.86)$ $(\pi/2, 1)$, $(2\pi/3, 0.86)$, $(2\pi/4, 0.71)$, $(5\pi/6, 0.5)$, $(\pi, 0)$, $(7\pi/6, -0.5)$, $(5\pi/4, -0.71)$, $(4\pi/3, -0.86)$ $(3\pi/3, -0.86)$, $(3\pi/2, -1)$, $(3\pi/3, -0.86)$, $(7\pi/4, -0.71)$, $(11\pi/6, -0.5)$ and $(2\pi, 0)$ in the xy-plane and join them by a free curve to obtain the curve y = sin x *i.e.*, the graph of $f(x)$ = sin x in the interval $[0, 2\pi]$ as shown in figure.



Graph of $f(x) = \sin x$ in $[0, 2x]$

As $f(x) = \sin x$ is a periodic function with period 2π. The graph of $f(x) = \sin x$ in the interval $[-2\pi, 0]$ is identical to its graph in $[0, 2\pi]$ as shown in figure.



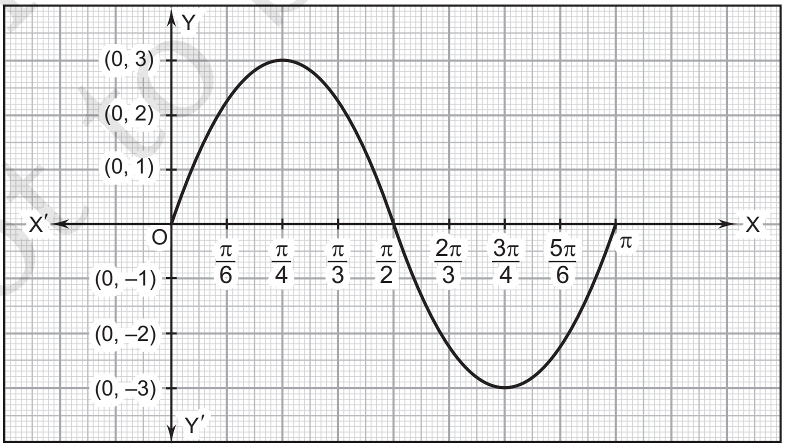
Graph of $f(x) = \sin x$ in $[-2\pi, 2\pi]$

Example 1. *Sketch the graph of the function $f(x) = 3 \sin 2x$.*

Solution. We know that $g(x) = \sin x$ is periodic function with period 2π. Therefore $f(x) = 3 \sin 2x$ is periodic funtion with period p. So, we will draw the graph of $f(x) = 3 \sin 2x$ is periodic function with period π. So, we will draw the graph of $f(x) = 3 \sin 2x$ in the interval $[0, \pi]$ and to draw (or know) its graph in other intervals such as $[-\pi, 0]$, $[\pi, 2\pi]$ etc. You may use the periodicity of the function. The value of $f(x) = 3 \sin 2x$ at various points in $(0, x)$ are listed in the following table.

x	0	$\frac{\pi}{12}$	$\frac{\pi}{8}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{3\pi}{8}$	$\frac{5\pi}{12}$
	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{5\pi}{8}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{7\pi}{8}$	$\frac{11\pi}{12}$
$2x$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$
	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$
$y = \sin 2x$	0	$\frac{3}{2}$ =1.5	$\frac{3}{\sqrt{2}}$ = 2.1	$\frac{3\sqrt{3}}{2}$ = 2.59	3	$\frac{3\sqrt{3}}{2}$ = 2.58	$\frac{3}{\sqrt{2}}$ = 2.1	$\frac{3}{2}$ = 1.5
	0	$-\frac{3}{2}$ = –1.5	$\frac{-3}{\sqrt{2}}$ = –2.1	$\frac{-3\sqrt{3}}{2}$ = –2.58	–3	$\frac{-3\sqrt{3}}{2}$ = –2.58	$\frac{-3}{\sqrt{2}}$ = –2.1	$\frac{3}{2}$ = –1.5

The points (0, 0) $(\pi/12, 1.5)$, $(\pi/8, 2.1)$, $(\pi/6, 2.58)$, $(\pi/4, 3)$, $(\pi/3, 2.58)$, $(3\pi/8, 2.13)$ $(5\pi/12, 1.5)$, $(\pi/2, 0)$, $(7\pi/12, -1.5)$, $(5\pi/8, -2.13)$, $(2\pi/3, -2.58)$ $(3\pi/4, -3)$, $(5\pi/6, -2.58)$, $(7\pi/8, -2.13)$, $(11\pi/12, -1.5)$ and $(\pi, 0)$ are plotted on a suitable scale in the xy-plane and joined by a free hand curve to obtain the graph of the function $f(x) = 3 \sin 2x$ i.e., the curve $y = 3 \sin 2x$ as shown in figure.



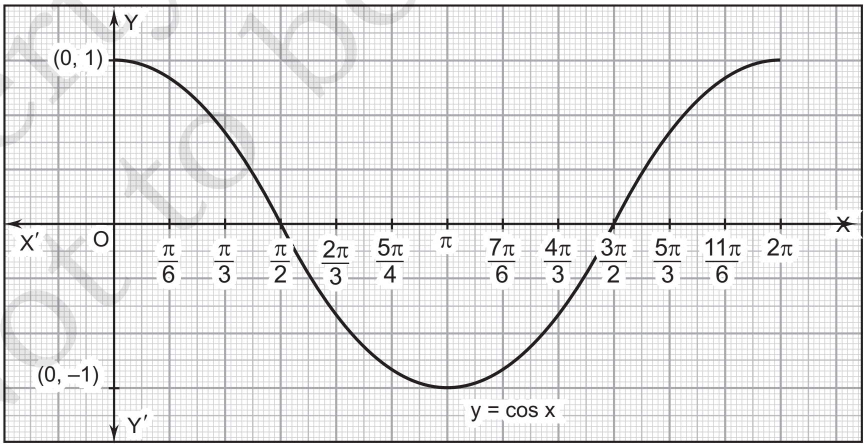
Graph of $f(x) = 3 \sin 2x$ in $[0, \pi]$

2. Graph of Cosine Function

We have learnt that $f(x)$ = cos x is a periodic function with period 2π. In order draw the graph of $f(x)$ = cos x it is sufficient to know its graph in $[0, 2\pi]$. The values of $f(x)$ = cos x at various points in $[0, 2\pi]$ are given in the following table.

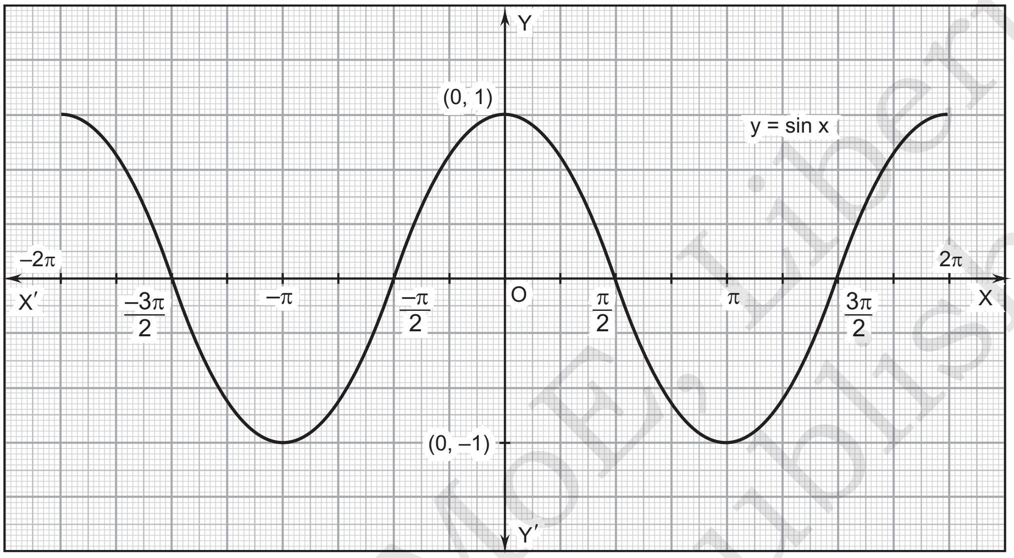
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	
	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
cos x	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	
	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	1

On a suitable scale, let us plot the points $(0, 1)$, $(\pi/6, \sqrt{3}/2)$, $(\pi/4, 1/\sqrt{2})$, $(\pi/3, 1/2)$, $(\pi/2, 0)$, $(2\pi/3, -1/2)$, $(3\pi/4, -1/\sqrt{2})$, $(5\pi/6, -\sqrt{3}/2)$, $(\pi, -1)$, $(7\pi/6, -\sqrt{3}/2)$, $(5\pi/4, -1/\sqrt{2})$, $(4\pi/3, -1/2)$, $(3\pi/2, 0)$, $(5\pi/3, 1/2)$, $(7\pi/4, 1/\sqrt{2})$, $(11\pi/6, \sqrt{3}/2)$, and $(2\pi, 1)$ in the xy-plane. Now join these points by a free hand curve to obtain the graph of the function $f(x)$ = cos x *i.e.* the curve y = cos x as shown in figure.



Graph of $f(x)$ = cos x, $0 \le x \le 2\pi$

The cosine function *i.e.* $f(x) = \cos x$ is an even function and the graph of an even function is symmetric about y-axis. So the graph of $f(x) = \cos x$ in $[-2\pi, 2\pi]$ is as shown in figure.



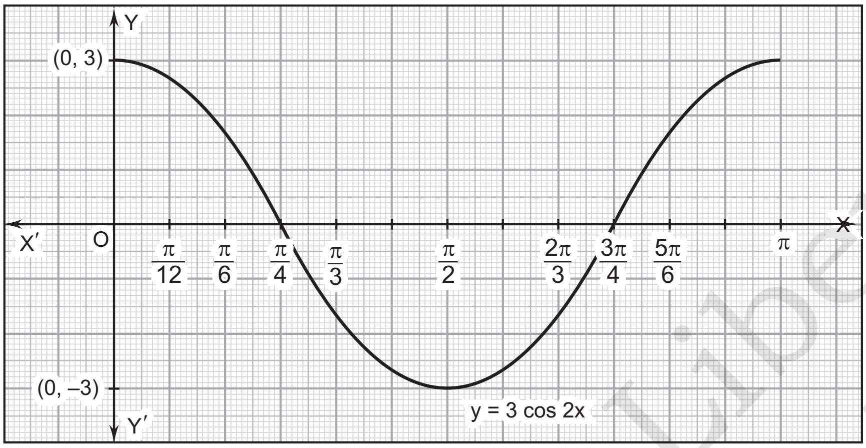
Graph of $f(x) = \cos x$, $-2\pi \le x \le 2\pi$.

Example 2. *Draw the graph of f(x) = 3 cos 2x.*

Solution. We know that cos x is a periodic function with period 2π. Therefore, $f(x) = 3\cos 2x$ is period with period π. So, it is sufficient to draw the graph of $f(x) = 3\cos 2x$ in the interval $[0, \pi]$. The values of $3\cos 2x$ for different values of x in $[0, x]$ are listed below.

x	0	$\frac{\pi}{12}$	$\frac{\pi}{8}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{3\pi}{8}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$
	$\frac{7\pi}{12}$	$\frac{5\pi}{8}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{7\pi}{8}$	$\frac{11\pi}{12}$	π	
$3\cos 2x$	3	$\frac{3\sqrt{3}}{2}$	$\frac{3}{\sqrt{2}}$	$\frac{3}{2}$	0	$-\frac{3}{2}$	$-\frac{3}{\sqrt{2}}$	$-\frac{3\sqrt{3}}{2}$	–3
	$-\frac{3\sqrt{3}}{2}$	$-\frac{3}{\sqrt{2}}$	$-\frac{3}{2}$	0	$-\frac{3}{2}$	$-\frac{3}{\sqrt{2}}$	$-\frac{3\sqrt{3}}{2}$	3	

Now, plot the points whose x-coordinates are points in the first row of the above table and the corresponding values in the second row as y-coordinates, By joining these points by a free hand curve, we obtain the graph of $f(x) = 3\cos 2x$ as shown in figure.



Graph of $f(x) = 3 \cos 2x$, $0 \le x \le \pi$

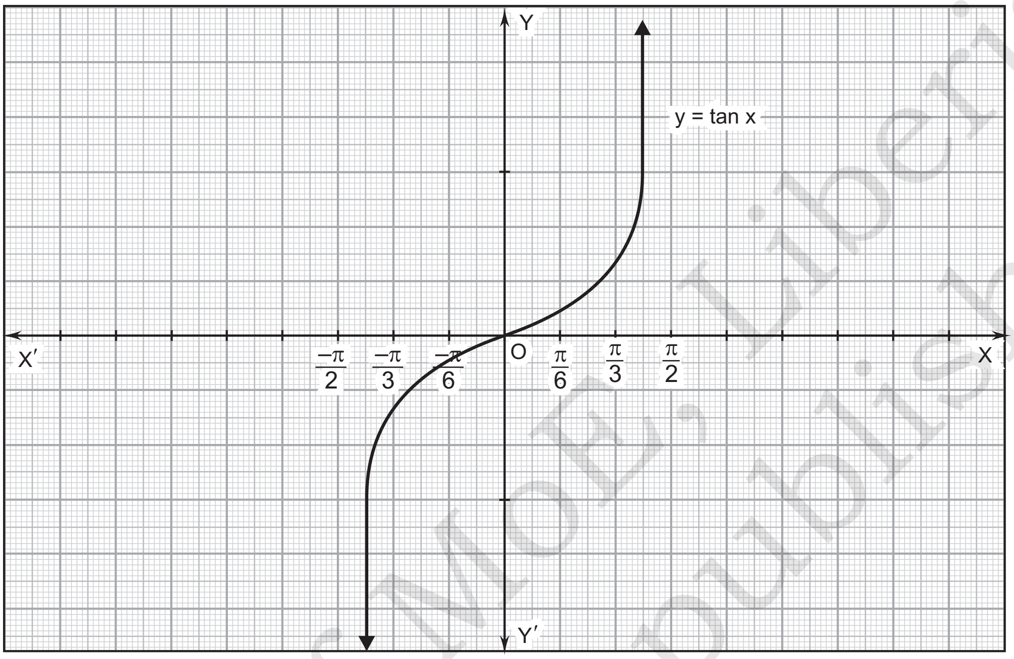
3. Graph of Tangent Function

The tangent function function *i.e.* $f(x) = \tan x$ is a periodic function with period π. So, it is sufficient to know the graph of $f(x) = \tan x$ over an interval of length π, in particular the interval $(-\pi/2, \pi/2)$. The values of $f(x) = \tan x$ at standard points in $(-\pi/2, 0)$ are negative of the corresponding values in $(0, \pi/2)$ and also listed in the following table:

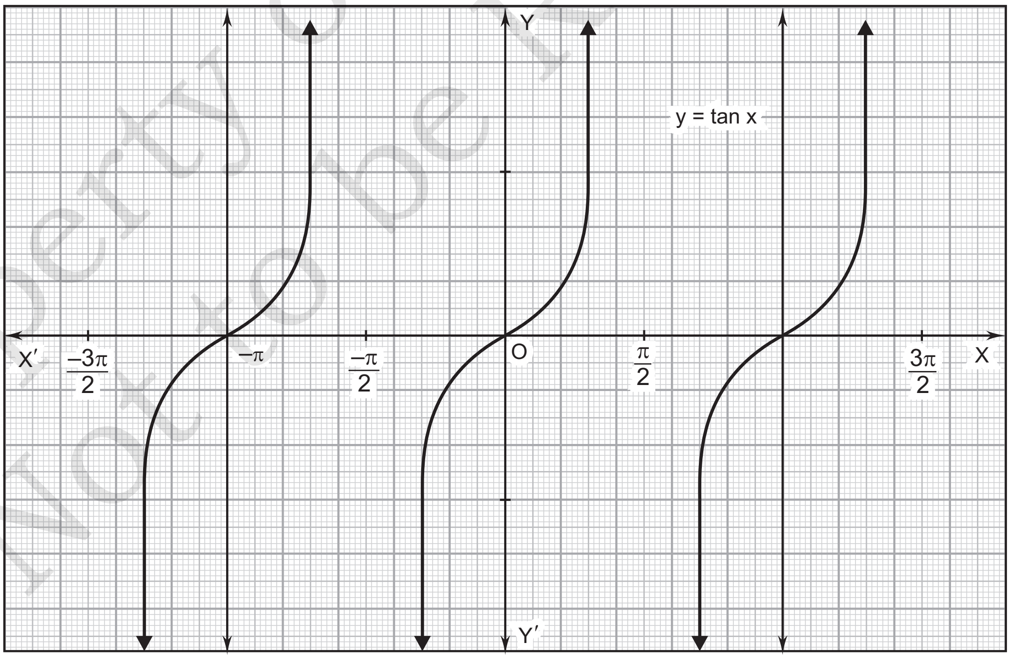
x	$-\frac{\pi}{2}$	$-\frac{5\pi}{12}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	$-\frac{\pi}{12}$	0
	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	
$f(x) = \tan x$	$-\infty$	$-\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	$-\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$	0
	$\frac{\sqrt{3}-1}{\sqrt{3}+1}$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\frac{\sqrt{3}+1}{\sqrt{3}-1}$	∞	

We also observe that $\tan x$ is an increasing function in $(0, \pi/2)$ and as $x \to \frac{\pi}{2}$ from the left the values of $f(x) = \tan x$ tend to infinity. So, the curve $y = \tan x$ gets closer and closer to the line values of $f(x) = \tan x$ tend to infinity. So, the curve $y = \tan x$ gets closer and closer to the line $x = \frac{\pi}{2}$ as $\to \frac{\pi}{2}$ from the left but it never touches the line $x = \frac{\pi}{2}$. The graph of $f(x) = \tan x$ is symmetric in opposite quadrants as the function is an add function. By plotting the points $(\pi/3, -\sqrt{3})$ $(-\pi/4, -1)$,

$(-\pi/6, -1/\sqrt{3})$, $(0, 0)$ $(\pi/6, 1/\sqrt{3})$, $(\pi/4, 1)$, $(\pi/3, \sqrt{3})$ and joining by a free hand curve, we obtain the sketch of the curve $y = \tan x$ as shown in figure.



Graph of $f(x) = \tan x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$



Graph of $f(x) = \tan x$, $-\frac{3\pi}{2} < x < \frac{3\pi}{2}$

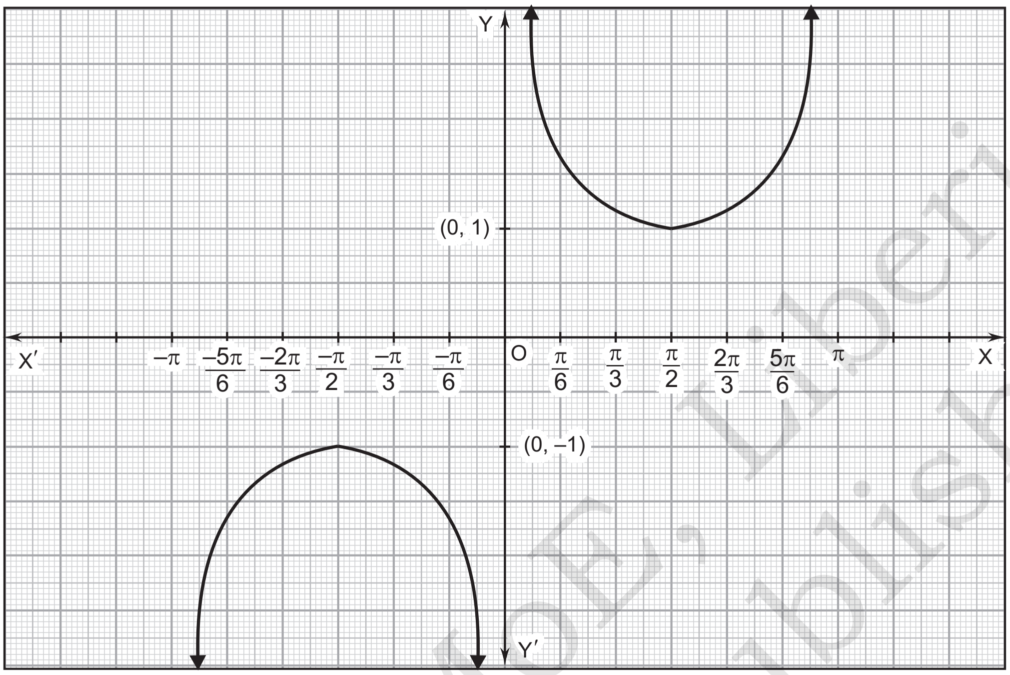
The function $f(x)$ = tan x is a periodic function with period π. So, the graph of $f(x)$ = tan x on (π/2, 3π/2) and (–3π/2, –π/2) is same as its graph on (–π/2, π/2) as shown in figure.

4. Graph of Cosecant Function

The cosecant function is the reciprocal of the sine function which is periodic with period 2π. So, $f(x)$ = cosec x is periodic with period 2π. Also, $f(x)$ is defined for all $x \in$ R – {$n\pi$; $n \in$ Z). In order to know about the graph of $f(x)$ = cosec x, it is sufficient to draw it on an interval of length 2π. Let us choose [0, 2π] as interval. The values of $f(x)$ = – cosec x at some standard points in [0, 2π] are listed in the following table. We observe that when x is close to zero or π in (0, π) the values of $f(x)$ tend to infinity. When $x \to \pi$ or $x \to 2\pi$ in (π, 2π) the values of $f(x) \to -\infty$.

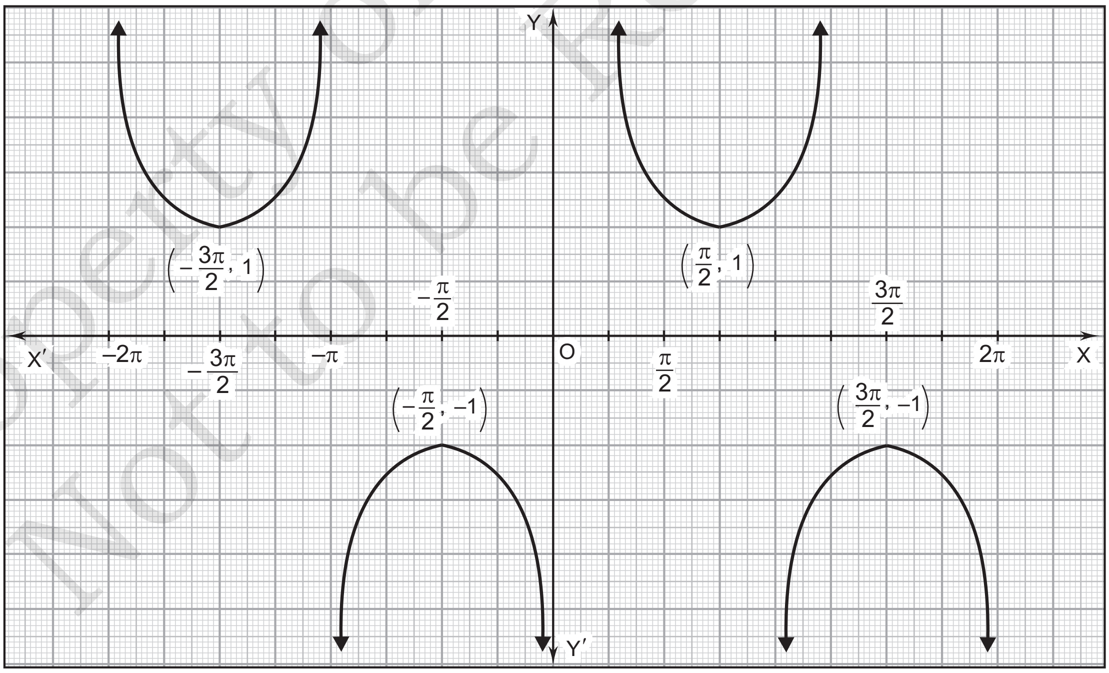
x	0^+	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$
	$\frac{5\pi}{6}$	π^-	π^+	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$
$f(x)$ = cosec x	$\to\infty$	2	$\sqrt{2}$ =1.41	$\frac{2}{\sqrt{3}}$ =1.5	1 =1.15	$\frac{2}{\sqrt{3}}$ =1.41	$\sqrt{2}$
	2	$\to\infty$	$\to-\infty$	–2	$-\sqrt{2}$ =–1.41	–1.15	–1

By plotting points (π/6, 2), (π/4, $\sqrt{2}$), (π/3, $2\sqrt{3}$), (π/2, 1), (2π/3, $2\sqrt{3}$), (3π/4, $\sqrt{2}$), (5π/6, 2), (7π/6, –2), (5π/4, $-\sqrt{2}$), (4π/3, $-2\sqrt{3}$), (3π/2, –1) (5π/3, $-2\sqrt{3}$), (7π/4, $-\sqrt{2}$), (11π/6, –2) and following these observations, we obtain the graph of the function $f(x)$ = cosec x *i.e.* the curve y = cosec x as shown in figure.



Graph of $f(x)$ = cosec x, $-\pi < x < \pi$, $x \neq 0$

The function $f(x)$ = cosec x is a periodic function with periodic 2π. So, the graph of $f(x)$ = cosec x in the travel $[-2\pi, 2\pi]$ is as shown in figure.

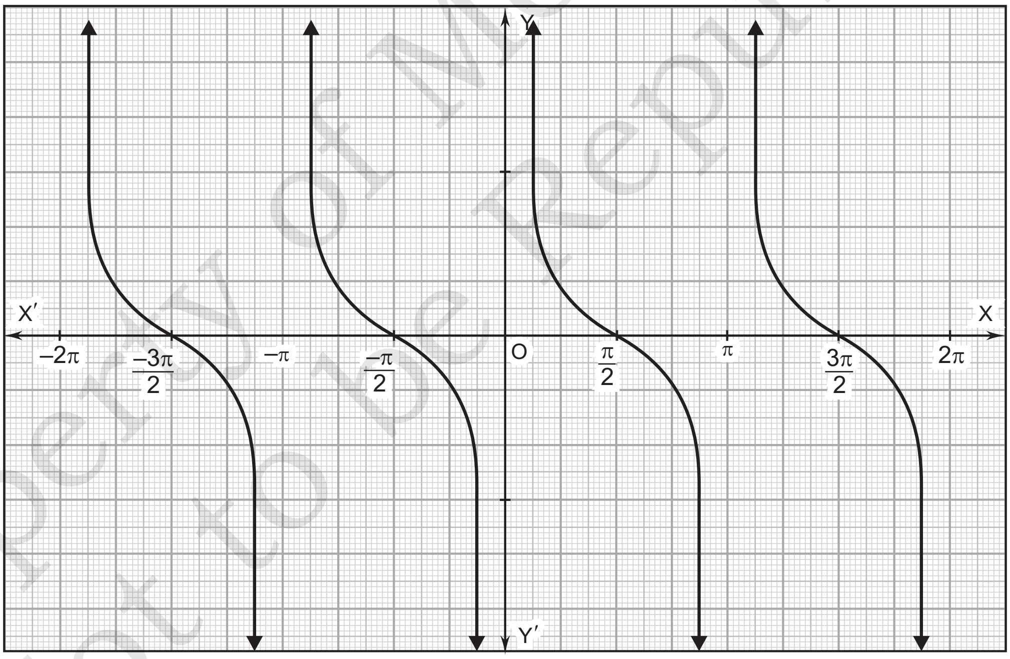


Graph of y = cosec x, $-2\pi < x < 2\pi$

5. Graph of Contagent Functions

We have learnt that the cotangent function i.e. $f(x)$ = cot x is a periodic function with period π. So, it is sufficient to know the graph of $f(x)$ = cot x over an interval of length π, in particular the interval $(0, \pi)$. The values of $f(x)$ = cot x at some standard values of x in $(0, \pi)$ are listed in the following table. We also observe the cot x is decreasing function in $(0, \pi)$ and as $x \to 0^-$ the values of cot $x \to +\infty$. So, the curve y = cot x gets closer and closer to the line $x = 0$ i.e. y-axis as $x \to 0^+$. We also observe that cot $x \to -\infty$ as $x \to \pi^-$ which means that the curve y = cot x gets closer and closer to the line $x = \pi$ as $x \to \pi$ from left hand side.

By plotting the values of $f(x)$ = cot x at various points in $(0, x)$ and keeping in mind the above observations, we obtain the graph of $f(x)$ = cot x in $(0, \pi)$ as shown in figure. As $f(x)$ = cot x is periodic function with period π so the graphs of $f(x)$ = cot x in $(-2\pi, -\pi)$, $(-\pi, 0)$ and $(\pi, 2\pi)$ are similar to the curve y = cot x in $(0, \pi)$ as shown in figure.



Graph of y = cot x, $-2\pi < x < 2\pi$

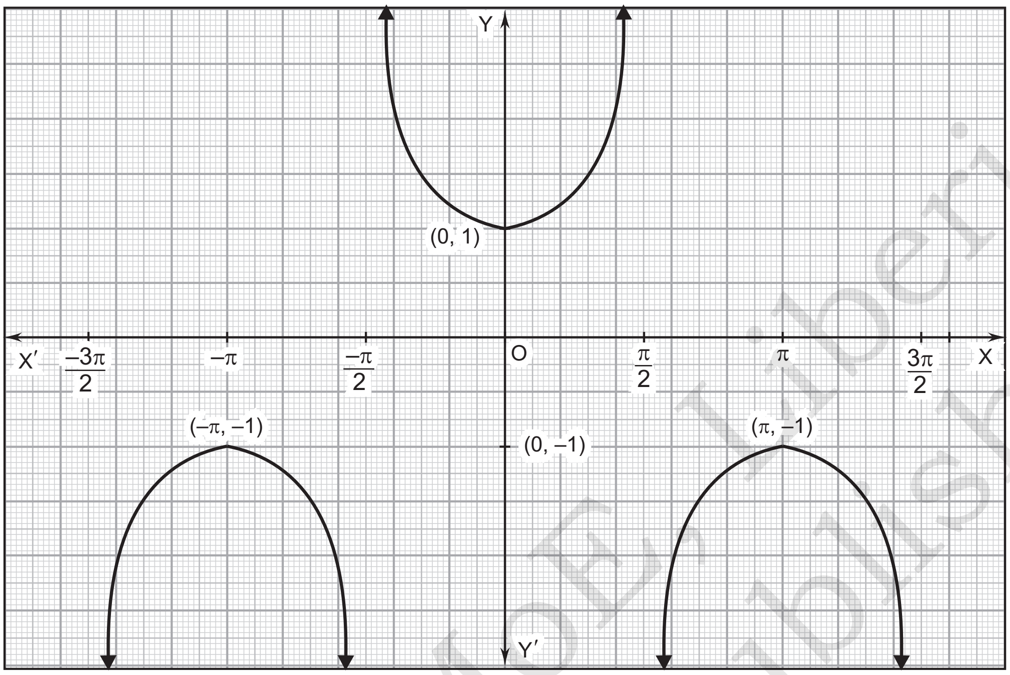
x	0^+	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π^-
	π^+	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$y = \cos x$	$\to\infty$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	$-\infty$
	$+\infty$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	$-\infty$

6. Graph of Secant Function

Similar to the other trigonometric functions the secant function is also a periodic function with period π. In order to know that graph of the secant function i.e. $f(x) = \sec x$, it is sufficient to draw it in an interval of length π, in particular the interval $(-\pi/2, \pi/2)$. We observe that the value of $f(x)$ tend to infinity as $x \to -\pi/2$ from right hand side. So, the graph of $f(x) = \sec x$ come closer and closer to $x = -\pi/2$ and $x = \pi/2$ but it never touches them. The value of $f(x) = \sec x$ at some standard points in $(-\pi/2, \pi/2)$ are listed in the following table.

x	$-\frac{\pi^+}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$
	$\frac{\pi}{3}$	$\frac{\pi^-}{2}$	$\frac{\pi^+}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$f(x) = \sec x$	$\to\infty$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$
	2	∞	$-\infty$	-2	$-\sqrt{2}$	$-\frac{2}{\sqrt{3}}$	-1

By plotting the points given by the above table and joining them by a free hand curve. We obtains the graph of $f(x) = \sec x$ *i.e.*, the curve $y = \sec x$ as shown in figure.



Graph of $y = \sec x,\ -\frac{3\pi}{2} < x < \frac{3\pi}{2}$

EXERCISE

1. Sketch the graphic of the function $f(x) = 3\sin\left(2x - \frac{\pi}{4}\right)$.

2. Draw the graph of $y = 3\cos 2x$.

3. Draw the graph of $f(x) = \cos\left(2x - \frac{\pi}{4}\right)$.